INFLUENCE OF THE MAGNETIC FIELD MAGNITUDE ON α PARTICLE DECELERATION

I. M. Gaisinskii, A. V. Kalinin, and A. E. Stepanov

UDC 533.95:538.4+533.9:621.039.61

Within the past 10-15 years a new direction, laser thermonuclear fusion (LTF), has opened up and been developed in the problem of controlled thermonuclear fusion. In this connection, a large quantity of research governing the requirements on the construction of the LTF reaction chambers has appeared. The concept of a reactor with the first wall shielded by a magnetic field has attracted the attention of researchers. In contrast to the traditional reactor scheme with magnetic containment of the plasma, where a high magnetic pressure is produced by strong fields, the field is used here to decelerate the highly energetic α particles to thermal velocities up to the time they reach the inner chamber wall. The conception of a LTF reactor with a "magnetic wall" was first proposed in [1], where the authors proposed the replacement of the already traditional LTF chamber of spherical geometry by a cylinder in which the field will be produced by a solenoid installed along the cylindrical part. Despite the attractiveness of this construction, the physical processes associated with the interaction mechanism of thermonuclear α particles escaping from a dense nucleus and an axial magnetic field have not been studied adequately.

1. STRUCTURE OF THE ESCAPING CHARGED PARTICLE FRONT AND ITS ELECTRICAL CONDUCTIVITY

In a thermonuclear microexplosion (of 7 mJ energy), α particles with the mean energy 3.5 meV are liberated [we consider them to be decelerated negligibly by escaping from the dense nucleus of the reaction zone (the worst case for the first reactor wall) while retaining an energy on the order of 2-3 meV]. We shall henceforth consider their spectrum to be δ-like. Consequently, the thermonuclear combustion time is short ($\tau_t \sim 10^{-11}$ sec), the first escaping α particles will terminate combustion in just 10^{-2} sec. In order to cancel the α particle current, an electron current should flow in the plasma. As the α particle escapes from the target, its "corona" surrounding the hot nucleus will emit electrons. It is important to estimate the scale of charge λ separation in a system consisting of α particles and electrons. A simple estimate shows that the value of λ (under the condition that $\lambda \ll R$, where R is the radius of the leading front of the α particles) is related to the energy included in the electrostatic field as follows:

$$W = 2\pi q_\alpha N_\alpha (\lambda/R)^2,$$

where q_α is the α particle charge, and W is the field energy.

If we consider all the α particle energy to be transformed into field energy, then even then it turns out that $\lambda \ll 10^{-2}$ cm up to distances on the order of several meters. Actually, the electron lag is due to their inertia and the friction of the "corona." Because of the smallness of the ratio m_e/M_α, the inertial lag is insignificant, $\lambda \sim 8 \cdot 10^{-6}$ cm. Because of the friction of the electrons of the "corona" the separation parameter is also small

$$\lambda^* = \frac{m_e v_e}{4 n_\alpha (R) e^2 \tau_{ee}} \sim 8 \cdot 10^{-8} R,$$

where v_e is the thermal velocity of the "corona" electrons, $n_\alpha(R)$ is the α particle density, and τ_{ee} is the electron collision time. The following "corona" parameters are taken for the estimate: $T_e \sim 10$ keV, $n_e \sim 10^{20}$ cm^{-3}.

Therefore, the escaping α particles are propagated in a thin layer of thickness $\delta \sim 10^{-2}$ cm whose charge is canceled while the scale of charge separation is $\lambda \ll \delta$ for plasmoid sizes on the order of the size of the reactor chamber (R ~ 1-3 m). It can be seen that the magnetic field lines of force turn out to be frozen in the plasma sheath. For this it is sufficient to note that the characteristic time of field diffusion into a plasma [2] is $\tau_m = 4\sigma L^2 c^{-2}$ (L is

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 40-46, November-December, 1979. Original article submitted November 27, 1978.

the scale of the magnetic field change in the sheath, $L \sim \delta$) is much greater than the characteristic time of development of the dispersion process $T_x = R_k/v_{\alpha,0} \sim 10^{-7}$ sec, where R_k is the radius of the first reactor wall, $v_{\alpha,0}$ is the initial velocity of α particle front dispersion corresponding to energy on the order of 2 meV. Indeed, if the plasma conductivity is considered classical, then $T_x \ll \tau_m \sim 10^{-5}$ sec.

2. MHD MODEL OF THE PROCESS OF PLASMA SHEATH DISPERSION IN AN AXIAL MAGNETIC FIELD

Joulean heat dissipation in a plasma sphere extending from the center of the reaction to the chamber walls can be neglected in deriving the initial system of equations since it is small compared to the plasmoid kinetic energy $E_k = N_\alpha M_\alpha v_{\alpha,0}^2/2$. Indeed, since

$$E_J \simeq \int_0^{T_x} \iiint_{D(t)} \frac{|\mathbf{j}(t)|^2}{\sigma(t)} dV dt, \quad \mathbf{j} = \frac{c}{4\pi}(\nabla \times \mathbf{B}) \simeq \frac{cB_0}{4\pi\delta}$$

($D(t)$ is the plasmoid volume),

then $E_J \sim 5.3$ kJ for $R(t) = R_k \sim 3$ m (R_k is the radius of the first wall of the cylindrical part of the reactor). Then the system of MHD equations under the condition that the magnetic field lines of force are frozen has the form

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \rho \mathbf{v} = 0,$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\nabla \frac{\mathbf{B}^2}{8\pi} + \frac{1}{4\pi} \mathbf{B} \cdot \nabla \cdot \mathbf{B}, \quad \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\nabla \times \mathbf{B}).$$

Potential conditions

$$\operatorname{div} \mathbf{B} = 0, \ \operatorname{rot} \mathbf{B} = 0$$

are valid for the field in the solenoid plasma ring-turn gap.

Taking account of the smallness of the plasmoid sheath thickness, and going over to spherical coordinates, we finally obtain the following system of equations

$$\frac{\partial N_\sigma}{\partial t} + \frac{N_\sigma}{R}\left[2\frac{\partial R}{\partial t} + \frac{\partial R}{\partial \theta}\frac{\partial \theta}{\partial t}\right] + \frac{\partial \theta}{\partial t}\left[\frac{\partial N_\sigma}{\partial \theta} + N_\sigma \operatorname{ctg}\theta\right] + \frac{\partial}{\partial \theta}\left(\frac{\partial \theta}{\partial t}\right) N_\sigma = 0,$$

$$\frac{\partial^2 R}{\partial t^2} = R\left(\frac{\partial \theta}{\partial t}\right)^2 - \frac{|\mathbf{B}(R, \theta, t)|^2}{8\pi M_\alpha N_\sigma}\cos\vartheta, \quad \frac{\partial^2 \theta}{\partial t^2} = -\left\{\frac{2}{R}\frac{\partial R}{\partial t}\frac{\partial \theta}{\partial t} + \frac{|\mathbf{B}(R, \theta, t)|^2}{8\pi M_\alpha N_\sigma R}\sin\vartheta\right\},$$

where $N_\sigma(\theta, t) = N(\theta, t)\delta(\theta, t)$ is the particle surface density, $N(\theta, t)\delta(\theta, t)$ are the particle density and plasma sheath thickness, respectively, and ϑ is the angle between the radius vector and the normal to the plasmoid surface defined by the expression

$$\vartheta = \arccos\left[1 + \left(\frac{1}{R}\frac{\partial R}{\partial \theta}\right)^2\right]^{-1/2}.$$

Solving the equation for the magnetic field potential by separation of variables, we obtain that the magnetic field components $\mathbf{B}(r, \theta, t)$ are

$$B_r = B_0(t)\left\{1 - \sum_{k=1}^{\infty}\left(\frac{R}{r}\right)^{k+2}(k+1)\frac{P_k(\cos\theta)}{\cos\theta}\Gamma_k\right\}\cos\theta,$$

$$B_\theta = -B_0(t)\left\{1 + \sum_{k=1}^{\infty}\left(\frac{R}{r}\right)^{k+2}\frac{dP_k(\cos\theta)}{d\cos\theta}\Gamma_k\right\}\sin\theta, \quad B_\varphi = 0,$$

where $P_k(\cos\theta)$ are Legendre polynomials, and the coefficients Γ_k are found from the condition of orthogonality of the field lines of force to the normal to the plasmoid at its surface

$$R\left[1 - \sum_{k=1}^{\infty}(k+1)\frac{P_k(\cos\theta)}{\cos\theta}\Gamma_k\right]\cos\theta + \frac{\partial R}{\partial \theta}\left[1 + \sum_{k=1}^{\infty}\frac{dP_k(\cos\theta)}{d\cos\theta}\Gamma_k\right]\sin\theta = 0.$$

If we limit outselves to the first terms of the expansion of the field components in spherical harmonics, we then obtain the magnetic dipole which was considered in [3] in an investigation of the expansion of a superconducting plasma cloud in an axial field. As he

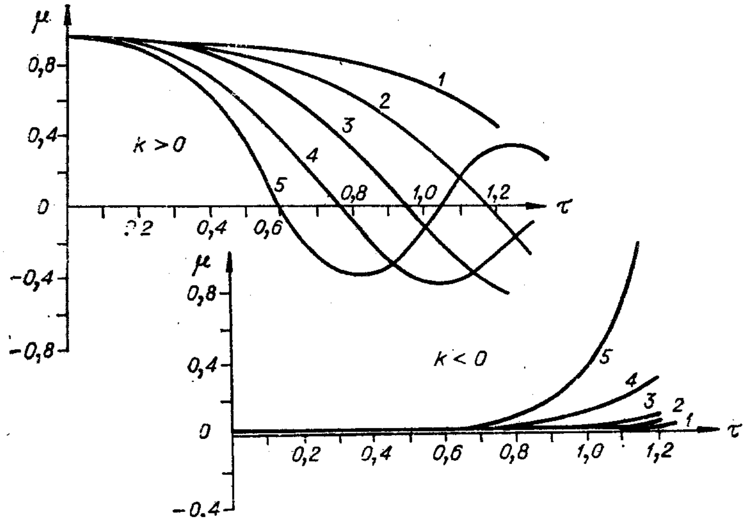

Fig. 1

himself showed, the model the author described is applicable everywhere except in that stage where the plasmoid starts to lose its sphericity. In order to close the system of equations, the magnetic field induction $B_0(t)$ must be expressed in terms of the magnetic flux $\Phi(t)$ in the absence of a plasmoid, and the equation for the electromagnetic induction is written for the outer loop.

Since

$$\Phi(t) = \int_{R(\pi/2,t)}^{R_t} \left|\mathbf{B}\left(r, \frac{\pi}{2}, t\right)\right| r\,dr,$$

then

$$B_0(t) = \frac{\Phi(t)}{\pi\left\{R_t^2 - R^2\left(\frac{\pi}{2}, t\right) + \sum_{k=1}^{\infty} R^2\left(\frac{\pi}{2}, t\right)\Gamma_k \frac{1}{k}\frac{dP_k(\cos\theta)}{d\cos\theta}\bigg|_{\theta=\frac{\pi}{2}}\left[1 - \left\{\frac{R\left(\frac{\pi}{2}, t\right)}{R_t}\right\}^k\right]\right\}},$$

where R_t is the radius of a solenoid turn. The electromagnetic induction equation has the form

$$\frac{d\Phi}{dt} = -\frac{R^*}{L}\frac{\Phi(t)}{1 - \left[R\left(\frac{\pi}{2}, t\right)\Big/R_t\right]^2}.$$

The following are the initial conditions for the equations:

$$R(\theta, t)\big|_{t=0} = R_0, \quad \frac{\partial R}{\partial t}\bigg|_{t=0} = v_{\alpha,0}, \quad N_\sigma(\theta, t)\bigg|_{t=0} = N_0\delta = N_{\sigma,0},$$

$$\frac{\partial\theta}{\partial t}\bigg|_{t=0} = 0, \quad \Phi\big|_{t=0} = \pi R_t^2 B_0 = \Phi_0,$$

where B_0 is the external solenoid-produced magnetic field induction, R^*, L are the resistance and inductance of the outer loop, $N_0 = N_\alpha/4\pi R_0^2\delta$ is the initial density of the plasma sheath, and R_0 is its initial radius.

3. QUESTIONS ASSOCIATED WITH THE DEVELOPMENT OF RAYLEIGH–TAYLOR INSTABILITY

The investigation of these phenomena are interesting since the surface density N_σ can diminish strongly because of sharp inflections during the development of plasma sheath surface perturbations, whereupon the magnetic field leaks through it until it succeeds in being retarded sufficiently.

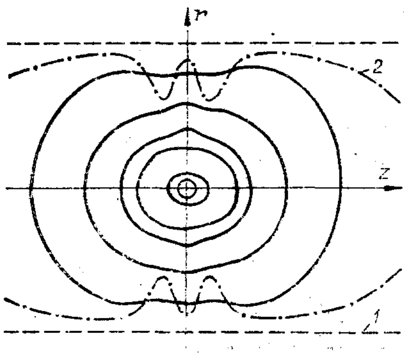

Fig. 2

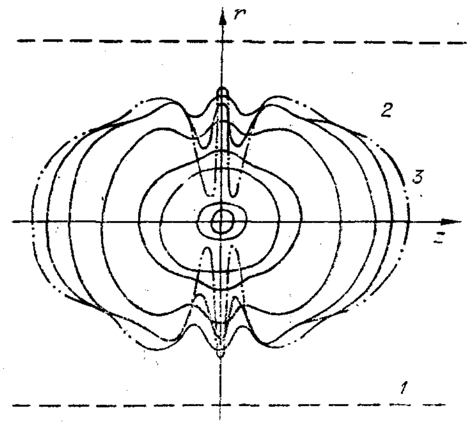

Fig. 3

Let us first consider the possibility of the development of hydromagnetic instabilities of Zhelobkov type in an equatorial section of the plasmoid (the r, φ plane in cylindrical coordinates). According to linear theory [4], any plasmoid surface perturbations in this section will be unstable; their increment is defined as follows:

$$\gamma = \frac{1}{4}\sqrt{\frac{2}{\pi}}B_0 (M_\alpha N_\sigma)^{-1/2}\sqrt{k},$$

where k is the wave number. It is seen from this expression that perturbations with modes $k \geq 10$ suceed in being developed sufficiently in the time $t \leq T_x$. However, linear theory yields no answer to the question of whether growing perturbations result in rupture of the plasma sheath. The method proposed in [5, 6], where the nonlinear mechanism of Taylor instability development upon the collapse of a thin-walled cylindrical liner is considered, can be used for a more detailed investigation.

In our case, the plasmoid dispersion is substantially three-dimensional in nature; however, a qualitative analysis of the development of instability in the plasmoid equatorial plane can be performed in a cylindrically symmetric dispersion approximation.

The equation of motion will then have the form

$$\rho\frac{\partial^2 \mathbf{r}}{\partial t^2} = \frac{B^2(r, \psi, t)}{8\pi}\left(\mathbf{e}_z \times \frac{\partial \mathbf{r}}{\partial \psi}\right),$$

where ψ is the Lagrange coordinate, i.e., $dm = \rho d\psi$.

The substitution $Z = x + iy = r \exp(i\psi)$ reduces it to the form

$$\rho\frac{\partial^2 Z}{\partial t^2} = i\frac{B^2(Z, \psi, t)}{8\pi}\frac{\partial Z}{\partial \psi}.$$

The initial conditions are given as follows:

$$Z(\psi, 0) = \exp(i\psi) - \frac{\mu^*}{k}\exp(ik\psi), \quad \frac{\partial Z}{\partial t}(\psi, 0) = v_{\alpha,0}\left[\exp(i\psi) - \frac{\mu^*}{k}\exp(ik\psi)\right].$$

If the solution is sought in the form

$$Z(\psi, t) = R(t)\left[\exp(i\psi) - \frac{\mu(t)}{k}\exp(ik\psi)\right],$$

then we arrive at the system of equations

$$\frac{d^2R}{d\tau^2} = -aR\left(\frac{1}{1-R^2}\right)^2, \quad \frac{d^2\mu}{d\tau^2} + \frac{2}{R}\frac{dR}{d\tau}\frac{d\mu}{d\tau} + \frac{1-k}{R}\frac{d^2R}{d\tau^2} = 0$$

with the boundary conditions

$$R(0) = \frac{R_0}{R_B}, \quad \left.\frac{dR}{d\tau}\right|_{\tau=0} = \frac{R_K}{R_B}, \quad \mu(0) = \mu^*, \quad \left.\frac{d\mu}{d\tau}\right|_{\tau=0} = 0,$$

where $a = (B_0^2 T_x^2 R_t^2 / 4 M_\alpha N_{\sigma,0})\delta$, τ is the dimensionless time.

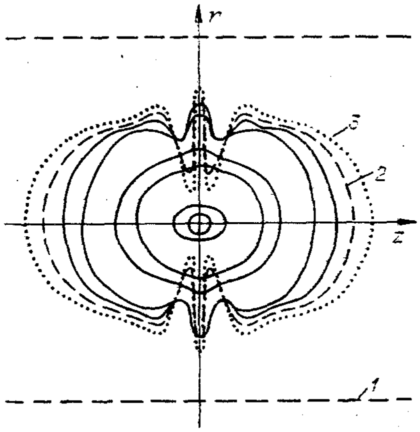

Fig. 4

For $|\mu(t^*)| \geq 1$ the curve enveloping the plasmoid equatorial section becomes reentrant, which is the beginning of sheath destruction. As a numerical analysis showed for $B_0 = 0.2$-0.7 T, $\mu^* = 0.95$ and positive k = 4, 8, 16, 32, 64 (curves 1-5 in Fig. 1, respectively), perturbation growth does not result in plasmoid destruction prior to its shutdown. For negative k the plasmoid is not destroyed only for sufficiently small initial perturbation amplitudes. Thus, for k = −64, it is necessary that μ^* not exceed 0.01.

The question of the origination of plasmoid surface perturbations is closely related to the prehistory of the process, the symmetry of compression and combustion of the target, hence it requries a considerably more complete examination, which is an independent problem.

Linear theory yields only a spectrum of the possible perturbations which succeed in being developed sufficiently during the time $t \leq T_x$ in an investigation of the possibility of a "sausage type" Rayleigh—Taylor instability (in the (r, z) plane of the cylindrical coordinates). It is determined from the condition

$$\frac{B_0^2 T_x^2}{32\pi M_\alpha N_\sigma}(9k - 8\delta k^2) \leqslant 1.$$

Hence, $k \leq 74$. In connection with the fact that it is not here possible to neglect the curvature of the magnetic field lines of force, a finer investigation is not performed successfully, however, initial plasmoid surface perturbations occur spontaneously right at the beginning of the process [7] because of the nonuniform distribution of the magnetic pressure P_M.

4. DISCUSSION OF THE RESULTS OF A NUMERICAL EXPERIMENT

The equations of the MHD model were solved numerically. The method of lines [8] was used. All the results presented are obtained for an initial partition in angles $\Delta\theta = 3°$. Computations were also executed with $\Delta\theta = 4.5$ and $6°$. The results hence did not change in any noticeable manner. The constraint imposed by the Courant condition was taken into account in selecting the time spacing.

The MHD-model equations were analyzed on a digital computer for the following parameters characteristic for this process $R_0 = 1$ cm, $R_k = 3$ m, $R_t = 3.25$ and 5 m (for a hybrid LTF reactor). The characteristic parameter governing the magnetic energy loss in the solenoid turns is $\omega = T_x \cdot R^*/L \sim 10^{-3}$ for $T_x \sim 10^{-7}$ sec. For $R_k/R_t = 0.6$, the analysis is performed for three versions of the initial value of the external magnetic field induction $B_0 = 0.5$, 0.75, 1.0 T (Figs. 2-4), and for $R_k/R_t = 0.925$ for two values $B_0 = 0.2$, 0.3 T (Figs. 5 and 6). The pattern of plasma sheath expansion at different times is shown in Figs. 2-6 (1 is the first wall of the cylindrical part of the reactor, 2 is the sheath geometry at the complete deceleration stage, 3 is the sheath geometry at the onset of its rupture, if it occurs). It is hence seen that the plasmoid loses its sphericity at a definite stage, which is completely natural since α particles encounter no resistance from the magnetic field in the direction of the axis of symmetry of the cylindrical part of the reactor. After a certain time t curvature of the sheath surface starts (valleys appear at the angles $70° < \theta < 90°$, $90° < \theta < 110°$, $250° < \theta < 270°$, $270° < \theta < 290°$), which can be considered as perturbations with the wave number k ~ 30-50. As is seen from Figs. 2-4, the fluctuations grow together with the magnitude of the

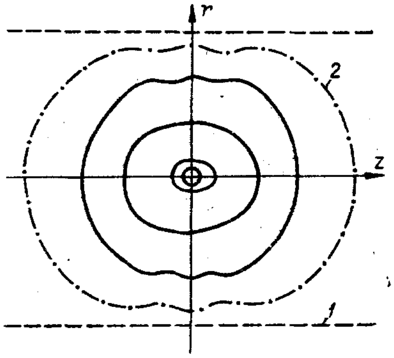

Fig. 5

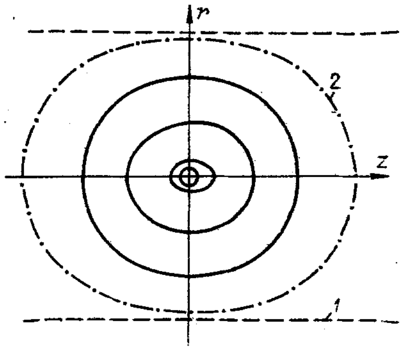

Fig. 6

initial external field induction and the plasmoid is destroyed for $B_0 \gtrsim 0.75$ T because of the abrupt diminution in the particle surface density $N_\sigma(\theta, t)$ at the sites of the inflections prior to the time of total deceleration.

As the ratio R_k/R_t increases to 0.925, a weaker field $B_0 \sim 0.2$–0.3 T is already required to shield the first reactor wall. The perturbations of the plasma sheath surface hence grow less intensively (see Figs. 5 and 6), and generally do not occur for $B_0 \leq 0.2$ T. This circumstance is important in the design of hybrid type reactors of cylindrical geometry with magnetic protection of the first wall since a field with induction $B_0 < 0.75$ T is incapable of decelerating thermonuclear α particle escaping from the reaction zone to thermal velocities for $R_k/R_t \leq 0.5$ T (the blanket occupies a large volume), but a stronger field will destroy a plasmoid long before it is totally decelerated.

LITERATURE CITED

1. T. Frank, D. Feidwald, T. Merson, and J. Devaney, "A laser fusion reactor concept utilizing magnetic fields for cavity wall protection," in: Proc. First Top. Meeting and the Technol. Control. Nuclear Fusion and Power Division, Am. Nucl. Soc., San Diego (1971).
2. N. Kroll and A. Tryvelpis, Principles of Plasma Physics [Russian translation], Mir, Moscow (1975).
3. Yu. P. Raizer, "On the deceleration and transformation of the energy of a plasma expanding into empty space in which there is a magnetic field," Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1963).
4. G. Knopfel, Superstrong Pulsed Magnetic Fields [Russian translation], Mir, Moscow (1972).
5. Yu. A. Vashilov and S. V. Pokrovskii, "Taylor instability of a thin cylindrical shell," Pisma Zh. Eksp. Teor. Fiz., 23, No. 8 (1976).
6. N. A. Inogamov, "Model analysis of Taylor instability of a shell," Pisma Zh. Tekh. Fiz., 3, No. 7 (1977).
7. L. M. Milne-Thompson, Theoretical Hydrodynamics, Crane Russak (1976).
8. V. I. Krylov, V. V. Bobkov, and P. I. Monastyrskii, Calculation Methods of Higher Mathematics [in Russian], Vol. 2, Vysshaya Shkola, Minsk (1972).